Scientific notations for the digital era
Version 1

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Reviewed and discussed at http://sjscience.org/article?id=527

Abstract Computers have profoundly changed the way scientific research is done. Whereas the importance of computers as research tools is evident to everyone, the impact of the digital revolution on the representation of scientific knowledge is not yet widely recognized. An ever increasing part of today’s scientific knowledge is expressed, published, and archived exclusively in the form of software and electronic datasets. In this essay, I compare these digital scientific notations to the traditional scientific notations that have been used for centuries, showing how the digital notations optimized for computerized processing are often an obstacle to scientific communication and to creative work by human scientists. I analyze the causes and propose guidelines for the design of more human-friendly digital scientific notations.
1 Introduction

Today’s computing culture is focused on results. Computers and software are seen primarily as tools that get a job done. They are judged by the utility of the results they produce, by the resources (mainly time and energy) they consume, and by the effort required for their construction and maintenance. In fact, we have the same utilitarian attitude towards computers and software as towards other technical artifacts such as refrigerators or airplanes.

In scientific research, however, the path that leads to a result is as important as the result itself. Drawing conclusions from an experimental measurement requires a good understanding of the experimental setup that was used to obtain the measurement. A scientist interpreting them must know the reliability and precision of the devices that were used, and be familiar with potential artifacts that could lead to misinterpretations. Likewise, the computational results obtained from scientific software can only be interpreted with a good understanding of what exactly the software does. Scientific software therefore has the same status in science as experimental setups and theoretical models.

In scientific discourse and in particular in the evaluation of a research publication, results are therefore scrutinized together with the path that lead to them. We expect experimentalists to explain the materials and methods they have used, and theoreticians to explain their reasoning in sufficient detail that their peers can understand it. We should thus treat computational science in the same way and require scientific software to be published and scrutinized in peer review. While publication of scientific software is slowly becoming common, peer review of this software remains exceptional. Its necessity is well recognized in principle, but the effort required for such a review is prohibitive. This is the most visible symptom of the problem that is the topic of this essay. More generally, this problem is that digital scientific knowledge is today expressed using notations such as programming languages, which are not suitable for communication between human scientists.

In the following, I will present a detailed analysis of this problem, and propose some general guidelines for improving the situation. The main audience is computational scientists, who have practical experience with doing science using computers but no formal training in computer science or in scientific epistemology. Readers with a computer science background may skim over much of the second part. Note that I will not propose the solution to the problem, nor even a solution. My goal is to convince computational scientists that there is a problem, and that it can be solved. Finding solutions that work well is likely to require many years and the participation of many people willing to test different ideas in practice.

The analysis that I present is applicable to all branches of science whose models are based on continuous mathematics, such as algebraic, differential, or integral equations. This includes almost all of physics and chemistry, a good part of
biology and the quantitative social sciences, and all domains of applied research that build on foundations in physics and chemistry. Much of what I say also applies to models based on discrete mathematics, such as graphs or cellular automata, but I will not consider them for the sake of simplicity. The examples I will use for illustration reflect my own background in computational biophysics, but readers shouldn’t find it difficult to substitute examples from their own field of work.

Outline

I start by summarizing the structure of scientific knowledge, explaining factual, procedural, and conceptual knowledge and why only the first two categories are used in computation. Next, I outline how scientific communication and the notations used for it evolved in the course of history. These two sections prepare the discussion of digital scientific knowledge and why we should care about it more than we do at the moment. This should provide sufficient motivation for the reader to work through the more technical section on formal languages, a well-known concept in computer science that unifies two categories that computational scientists tend to see as distinct: file formats for data and programming languages, the two dominant forms of digital scientific notation today.

After this more theoretical part, I explain the importance of composition of information items using as an example the simulation of celestial mechanics, with an emphasis on the constraints on the composition of digital knowledge. My goal is to illustrate what one should be able to do with a proper digital scientific notation. I then compare to the state of the art in computational science, pointing out how it is inadequate for the study of complex systems. One obstacle to improvement is a perceived dichotomy between software and data, which has its roots in computing technology but has no counterpart in the structure of scientific knowledge.

An important point that is often overlooked is the status of formal languages as the main human-computer interface in computational science. Doing research is a different task from developing software, and requires a different interface. In particular, we should pay more attention to the difference between human and computational semantics and to the need for simplicity and flexibility of a notation suitable for humans doing creative research. Moreover, a digital scientific notation must permit precise references to the scientific record.

In the last part of this essay, I consider solution strategies for the problems that I have identified. I show two examples of how formal languages can be made simple and flexible while providing a straightforward mechanism for composition: XML with its namespace mechanism for composition, and the Lisp family of programming languages with its macro system for creating small embedded formal languages. I conclude by proposing design guidelines for digital scientific notations.
2 The structure of scientific knowledge

For the following discussion of scientific notation, it is useful to classify scientific knowledge into three categories: factual, procedural, and conceptual knowledge. Factual knowledge consists of the kind of information one can record in tables, diagrams, or databases: the density of water, the names of the bones in the human body, the resolution of an instrument, etc. Procedural knowledge is about doing things, such as using a microscope or finding the integral of a function. Conceptual knowledge consists of principles, classifications, theories, and other means that people use to organize and reason about facts and actions.

Factual and procedural knowledge relies on conceptual knowledge. A table listing the density of water at different temperatures refers to the concepts of density and temperature. Instructions for using a microscope refer to concepts such as sample or focus. Interpreting factual or procedural knowledge requires a prior knowledge of the underlying conceptual knowledge.

Conceptual knowledge has a hierarchical structure, with the definition of every concept referring to more fundamental concepts. This leads to the question of where this recursive process ends, i.e. what the most fundamental concepts are. When considering human knowledge as a whole, this is a non-trivial problem in epistemology. For a discussion of scientific knowledge, and in particular for the present discussion of scientific notation, it is sufficient to consider the concepts of everyday life as a given base level.

Factual and procedural knowledge often refer to each other. The statement “The orbit of the Moon around the Earth is reproduced to precision A by solving Newton’s equations for the solar system using numerical algorithm B and initial values C” is factual knowledge, once specific A, B, and C are provided. But algorithm B is procedural knowledge, which in turn refers to some other factual knowledge, such as the masses of the Sun and its planets.

A final missing piece is metadata. Every piece of factual and procedural knowledge comes with information attached to it that describes its provenance and known limits of validity. A table showing the density of water at different temperatures should state how, when, under which conditions, and by who the listed values were obtained. It should also provide an estimate of the values’ accuracy.

In summary, the structure of scientific knowledge can be described as a web of factual and procedural knowledge items that refer to each other, and which are expressed in terms of concepts from the universe of conceptual knowledge. The latter consists of multiple layers, with concepts from everyday life at the bottom. Every layer refers to concepts from lower, more fundamental layers.
3 The evolution of scientific communication

Most of scientific communication takes place through research articles, which are narratives that propose new factual and procedural knowledge, occasionally also new concepts, and try to convince the scientific community of the pertinence of this information. Over time, as a given subject area becomes better understood, the scientific community usually reaches a consensus about which concepts are the most useful for describing its phenomena. The knowledge from a large number of research articles is then distilled into review articles, monographs, and other reference works. The knowledge considered most fundamental ends up in textbooks for transmission to the next generation of scientists. Each unit of scientific communication is written for a specific audience, and relies on a stack of conceptual layers that this audience is expected to be familiar with.

Before the use of computers, scientific knowledge was mainly recorded on paper, using three forms of notation: written language, images, and tables. Written text combines plain language, domain-specific vocabulary, and shorthand notation such as mathematical formulas. Images include both drawings and observations captured in photographs, radiographs, etc. Tables represent datasets, which are most often numerical.

There is a close relation between the conceptual knowledge on which a narrative relies and the notation that it employs. Domain-specific vocabulary directly names relevant concepts. Shorthand notation replaces frequently used words and lengthy sentences that involve these concepts. For example, Newton’s laws of motion are commonly written as

\[ F = m \cdot a \]

whose full-length equivalent is “The force acting on a point mass is equal to the product of its mass and its acceleration.” Force, mass, and acceleration are concepts from mechanics and \( F, m, \) and \( a \) are conventional shorthands for them. The symbols \( = \) and \( \cdot \) are shorthands for the concepts of equality and product, both of which come from more fundamental conceptual layers in mathematics.

The standardization of scientific notation is variable and usually related to the stability of the concepts that it expresses. Well-established conceptual layers come with a consensus notation, whereas young conceptual layers can be expressed very differently by different authors. Scientists “play around” with both the concepts and the notations in rapidly evolving fields, before eventually settling on a consensus that has proven to work well enough. Even the most basic aspects of mathematical notation that we take for granted today were at some time the subject of substantial tinkering [1]. Moreover, even a consensus notation is not completely rigid. Personal and disciplinary tastes and preferences are one cause of variation. As an example, there are several common notations for distinguishing vectors from scalars in geometry. Another cause is the limited
number of concise names and labels. For example, the preference for one-letter names in mathematical formulas, combined with the useful convention of each letter having only one meaning in a given document, often imposes deviations from consensus naming schemes.

This pattern of a high variability during innovation phases giving way to consensus as a field or technology matures is ubiquitous in science and engineering. The time scale of the consolidation process is often decisive for reaching a satisfactory consensus. The lack of consensus in mature technology is felt as a nuisance by its users. A good example is the pointlessness of chargers for mobile phones. On the other hand, premature consensus creates badly adapted technology that is difficult to get rid of. Computing technology is particularly affected by this problem. In fact, most of the standardized technology in computing – file formats, programming languages, file systems, Internet protocols, etc. – is no longer adequate for today’s requirements. The reason is that the technical possibilities – and, as a consequence, user demands – evolve too fast for an orderly consensus formation, whose time scale is defined by human cognitive processes and social interactions that, unlike technological progress, have not seen any spectacular acceleration.

4 Digital scientific knowledge

In the context of computing, factual knowledge is stored in datasets, whereas procedural knowledge takes the form of algorithms. Conceptual knowledge is not affected by the transition from manual to mechanized computation. Like research articles and reference tables, datasets and algorithms implicitly refer to conceptual knowledge to give meaning to their contents. However, the concepts are not explicitly represented in the computer, because they are not required to perform a computation. Applying algorithms to datasets is a purely mechanical operation that does not require any knowledge or understanding of the underlying concepts. What does require an understanding of the concepts is the verification that a given computation is scientifically meaningful.

It is of course possible to store and process conceptual knowledge using computers, e.g. in the form of ontologies, which represent conceptual knowledge as factual knowledge at a different semantic level. Such approaches are finding their place in scientific communication in the form of semantic publishing [2], whose goal is to make the scientific record machine-readable and thus accessible to automated analysis. However, performing a computation and managing information about this computation are different and independent operations, just like using a microscope is a different activity from researching the history of microscopy. I will come back to the role of digital scientific notations in semantic publishing later.

This dissociation of mechanical computation from the conceptual knowledge base
that defines its meaning has been recognized as a problem in various domains of
digital knowledge management, for example in database design [3]. The typical
symptom is the existence of electronic datasets that nobody can interpret any
more, because the original designers of the software and data formats did not
document their work sufficiently well for their colleagues and successors. A
frequent variant is people modifying software and data formats without updating
the documentation. Every computational scientist has probably experienced the
difficulties of dealing with datasets stored in undocumented formats, and with
software whose inner workings are not described anywhere in an understandable
form.

The most vicious manifestation of this problem relates to scientific software. Even
when software is developed respecting the best practices of software engineering,
it may nevertheless compute something else than what its users think it computes.
Documentation can help to some degree by explaining the authors’ intentions
to the users, but nothing permits to verify that the documentation is complete
and accurate. The only way to make sure that users understand what a piece
of software computes is making the software’s source code comprehensible for
human readers. Today, most scientific software source code is unintelligible to its
users, and sometimes it even becomes unintelligible to its developers over time.

Some of the problems we are observing in computational science today are direct
consequences of the fact that scientists have an insufficient understanding of
the software they use. In particular, it suffers from rampant software errors
[4, 5] and the near-universal non-reproducibility of computational results [6, 7].
The scientific community has failed so far to fully appreciate the double role
of scientific software as tools for performing computations and as repositories
of scientific knowledge [8]. It has uncritically adopted notations for digital
knowledge that are not adapted to human communication. As a consequence,
the all-important critical discourse that makes scientific research self-correcting
in the long run does not adequately cover digital scientific knowledge.

5 Formal languages

The defining characteristic of digital scientific knowledge is the use of formal
languages, rather then informal languages of human communication. The term
“formal language” is commonly used in computer science, but in computational
science we usually speak of “data formats”, “file formats”, and “programming
languages”, all of which are specific kinds of formal languages. In this section, I
will give a minimal overview of the characteristics of formal languages, which is
necessary for understanding their implications for digital scientific knowledge.

At the hardware level of a digital computer, a computation is a multi-step process
that transforms an input bit sequence into an output bit sequence. Information
processing by computers thus requires all data to be expressed as bit sequences.
Dealing with bit sequences is, however, very inconvenient for humans. We therefore use data representations that are more suitable for human brains, but still exactly convertible from and to the bit sequences that are stored in a computer’s memory. These representations are called formal languages. The definition of a formal language specifies precisely how some piece of information is encoded in sequences of bits. Many formal languages use text characters instead of bits for another level of convenience. Since the mapping from text characters to bit sequences is straightforward (the currently dominant mapping is called Unicode [9]), this makes little difference in practice.

The definition of a formal language consists of two parts, syntax and semantics. Syntax defines which bit patterns or text strings are valid data items in the language. Syntax rules can be verified by a program called a parser. Semantics define the meaning of syntactically correct data items. With one important exception, semantics are mere conventions for the interpretation of digital data. As I explained above, meaning refers to conceptual knowledge that a computer neither has nor needs, since all it does is process bit sequences. The exception concerns formal languages for expressing programs, i.e. the rules used by the computer for transforming data. The semantics of a programming language define how each operation transforms input data into output data. Writing down such transformation rules obviously requires a notation for the data that is being worked on. For that reason, a programming language also defines the syntax and semantics of data structures. In fact, a programming language can express all aspects of a computation. We use separate languages for data (“file formats”) only as a convenience for users and for improving the efficiency of our computers.

There is a huge number of formal languages today, which can be organized into a hierarchy of abstraction layers, such that languages at a higher level can incorporate languages from lower levels. As a simple example, a programming language such as Fortran incorporates formal languages defining individual data elements - integers, floating-point numbers, etc. At the lowest level of this hierarchy, close to the bit level at which computing hardware operates, we have formal languages such as Unicode [9] for text characters or the floating-point number formats of IEEE standard 754 [10]. One level up we find the memory layout of Fortran arrays, the layout of UTF-8 encoded text files, and many other basic data structures and file formats. Structured file formats such as XML [11] or HDF5 [12] are defined on the next higher level, as they incorporate basic data structures such as trees, arrays, or text strings. Programming languages such as Fortran or C reside on that level as well.

Defining the semantics of a programming language is not a straightforward task. For non-programming formal languages, semantics are mere conventions and therefore defined by a document written for human readers. The same approach can be adopted for a programming language, resulting for example in the C language standard [13]. But the semantics of programs also matter for their execution by a computer, and therefore a “computer-readable” definition of the semantics is required as well. It takes the form of either a program that translates
the programming language into processor instructions, called a compiler, or a program that directly performs the actions of the programming language, called an interpreter. We thus have the C language standard defining the semantics of the C language for human readers, and a C compiler defining the semantics for execution by the computer. Unfortunately, there is no way to ensure or verify that the two definitions are equivalent. A computer program cannot do it, because the C language standard is not written in a formal language. A human computing expert cannot do it reliably, because a C compiler is much too complicated for verification by inspection.

This is in fact the same situation as I described in the last section for scientific software: the compiler is the equivalent of the scientific software, and the language definition is the equivalent of its documentation. This is not just a superficial analogy: there is in fact no profound difference between a compiler and a piece of scientific software. Both transform input data into output data according to complex rules that are explained to human readers in a separate document. Compilers are executable implementations of programming languages in the same way as scientific software is an executable implementation of scientific models. This analogy is useful because computer scientists have invested considerable effort into bridging the gap between executable and human-readable specifications of programming languages. Most of the ideas and some of the tools developed in this process can thus be adapted to scientific software.

The basic idea is to introduce formal specifications, which are written in formal languages and thus computer-readable, but which simpler than the software whose behavior they specify, and therefore more comprehensible to human readers. Specifications are simpler than the actual software for several reasons. One of them is that a specification can neglect many usability issues of software: performance, use of resources, portability between platforms, user interfaces, etc. are all irrelevant for specifying the core computations that the software performs. More simplification is possible if one accepts mere definitions instead of algorithms. A definition allows to test if a result is correct, but is not sufficient to obtain a result. As a simple example, consider sorting. The definition of a sorted list of items is “an ordered list whose elements are the same as those of the input list”. Any algorithm for actually performing the sort operation is much more complicated. For a human reader, the definition is usually sufficient to understand what is going on, and testing procedures can verify that the algorithms implemented in software actually conform to the definition.

Like specification languages, formal languages for representing digital scientific knowledge must aim for simplicity to facilitate comprehension by human scientists, in particular those not directly involved with the development of scientific software. Much of the experience gained from work on specification languages can probably be applied in the design of formal languages for science, but there are also differences to be taken into account. In particular, scientific knowledge differs from software in that its principal purpose is not to compute something. Computation in science is a means to an end which is understanding nature. In
the next section, I will show a few examples of scientific information items and how they are used in the construction of scientific software while also serving different purposes.

6 Composition of information items

A key operation in information management is the composition of data from various sources into a more complex assembly. Composition is a well-known concept in software development, as software is usually assembled from building blocks (procedures, classes, modules, ...), including preexisting ones taken from libraries. But composition is also an everyday task in the theoretical sciences, even though it is not labeled as such and in fact rarely ever identified as a distinct activity.

6.1 Example: composing a model for the solar system

Suppose you want to predict the positions of the planets of our solar system over a few years. You would start with Newton’s 17th-century description of celestial mechanics and compose a model from the following ingredients:

1. Newton’s law of motion: $F = m \cdot a$
2. Newton’s law of gravitation: $F_{ij} = G \frac{m_i m_j}{|r_i - r_j|^2}$
3. The masses $m_i$ of the sun and the planets.
4. A set of parameters, derived from past astronomical observations, to define the initial state.

All these put together define the positions of the celestial bodies at all times in the past and future. But each of these items has a meaning independently of the others, and can be put to other uses, such as computing how fast an apple falls to the ground. You can also use the first two ingredients to prove energy conservation in celestial mechanics, or to derive Kepler’s laws. Moreover, each of these pieces comes from a different source (observation, theoretical hypothesis, ...) that requires a specific approach to validation. We want to be able to compose them into a new entity called “model for the solar system”, but we also want each piece to retain its own identity for other uses. Ideally, we want to present our solar system model as a composition that references the individual ingredients. And in the traditional printed-paper system of scientific communication, that’s exactly what we do.

Let’s move on to computation. To make an actual prediction, you have to add some more ingredients. The model as composed above only defines the planetary orbits, but doesn’t tell you how to compute them. So you need to add:
5. A numerical solver for ordinary differential equations (ODEs), such as Runge-Kutta.

6. Suitable parameters for that solver, depending on your accuracy and precision requirements. For Runge-Kutta, that’s the size of the integration time step.

7. A finite-size number representation with associated rules for arithmetic, because you can’t compute with real numbers.

You can then take a large stock of pencils and paper and start to compute. If you prefer to delegate the grunt work to a computer, you need one final ingredient:

8. A programming language, implemented in the form of a compiler or interpreter.

Your final composition is then a simulation program for celestial mechanics, made from eight distinct ingredients. Ideally, you would publish each ingredient and the composition separately as nine machine-readable nanopublications [14]. Unfortunately, with the current state of the art in computational science, that is not yet possible.

### 6.2 Composition of digital knowledge

In the pre-digital era, composition was never much of a problem. A scientist would take a few research articles or monographs describing the various ingredients, and then write down their composition on a fresh sheet of paper. Variations in the notations across different sources would be no more than an inconvenience. Our pre-digital scientist would translate notation into concepts when reading each source, and the concepts into his or her preferred notation when writing down the composition. As long as the concepts match, as they do in any mature field of science, that is routine work.

Composition of digital knowledge is very different. The items to be composed must be matched not only in terms of (human) concepts, but also in terms of the syntax and semantics of a formal language. And that means that all ingredients must be expressed in the same formal language, which is then also the language of the composed assembly.

If we start from ingredients expressed in different languages, we have basically two options: translate everything to a common language, or define a new formal language that is a superset of all the languages used for expressing the various ingredients. We can of course choose a mixture of these two extreme approaches. But both of them imply a lot of overhead and add considerable complexity to the composed assembly. Translation requires either tedious and error-prone manual labor, or writing a program to do the job. Defining a superlanguage requires implementing software tools for processing it.

As an illustration, consider a frequent situation in computational science: a data processing program that reads a specific file format, and a dataset stored in a different format. The translation option means writing a file format converter.
The superlanguage option means extending the data processing program to read a second file format. In both cases, the use of multiple formal languages adds complexity to the composition that is unrelated to the real problem to be solved, which is the data analysis. In software engineering, this is known as “accidental complexity”, as opposed to the “essential complexity” inherent in the task [15].

As a second example, consider writing a program that is supposed to call a procedure written in language A and another procedure written in language B. The translation option means writing a compiler from A to B or vice-versa. The superlanguage option means writing a compiler or interpreter that accepts both languages A and B. A mixed approach could use two compilers, one for A and one for B, that share a common target language. The latter solution seems easy at first sight, because compilers from A and B to processor instructions probably already exist. However, the target language of a compiler is not “processor instructions” but “the processor instruction set plus specific representations of data structures and conventions for code composition and memory management”. It is unlikely that two unrelated compilers for A and B have the same target language at this level of detail. Practice has shown that combining code written in different programming languages is always a source of trouble and errors, except when using tools that were explicitly designed from the start for implementing the superlanguage.

Many of the chores and frustrations in the daily life of a computational scientist are manifestations of the composition problem for digital knowledge. Some examples are

- file format conversion, as explained above
- combining code in different languages, also explained above
- software installation, which is the composition of an operating system with libraries and application-specific software into a functioning whole
- package management, which is an attempt to facilitate software installation that re-creates the problem it tries to solve at another level
- software maintenance, which is the continuous modification of source code to keep it composable with changing computational environments
- I/O code in scientific software, which handles the composition of software and input data into a completely specified computation
- workflow management, which is the composition of datasets with multiple independently written and installed software packages into a single computation

These examples should be sufficient to show that the management of composition must be a high-priority consideration when designing formal languages for digital scientific knowledge.
7 The state of the art in managing digital scientific knowledge

In the last section I have listed the ingredients that need to be combined in order to make a solar system simulator. Let’s look at how such a simulator is actually structured using today’s scientific computing technology. We have the following clearly identifiable pieces:

1. A simulation program, written in a programming language such as Fortran or C++, which incorporates ingredients 1, 2, 5, 7, and 8.

2. An input file for that program, written in a special-purpose formal language defined by the author of the simulation program, containing ingredients 3, 4, and 6.

The structure of the input file is usually simple, meaning that it is straightforward to isolate ingredients 3, 4, and 6 from it. There is even a good chance that the input file will permit annotation of these items, indicating the sources they were taken from. If we are really lucky, the formal language of the input file is documented and designed to permit the extraction of information for other uses.

The simulation program itself is almost certainly a monolithic piece of information that combines 1, 2, 5, 7, and 8 in an inextricable way. None of the ingredients is easy to identify by inspection, and we’d better not even envisage extracting them using computational tools for other uses. If we want to change something, e.g. use a different ODE solver or a different finite-size number representation, we’d probably rewrite large parts of the program from scratch. Worse, changing the finite-size number representation might actually force us to rewrite the program in a different language.

This is how today’s scientific software is typically written, but let’s also look at what we could do, using today’s technology, if we were making a special effort to maintain the modular structure of our knowledge assembly.

The easiest part to factor out is number 5, the ODE solver. We could use one from a program library, and even choose a library that proposes several solvers. But using such a library comes at an additional cost in combining all the parts. We have to write ingredients 1 and 2 according to the rules of the library, and accept for 7 and 8 whatever the library allows us to use. In fact, the library modifies the formal language we use for writing our software, adding features but also imposing constraints. Fortran plus ODEPACK is not the same language as Fortran on its own.

Superficially, we can also factor out ingredients 1 and 2, which define the equations fed to the ODE solver. We could isolate these ingredients in the form of procedures (also called subroutines or functions). But those procedures do not represent the original equations. They only represent one aspect of the equations: the numerical evaluation of some of their subterms. We could not
Finally, we could envisage factoring out ingredient 7, the number representation. For example, we could use a library such as MPFR [16] to get access to wide range of floating-point formats. But the same remark applies as for the use of an ODE library: we would have to translate everything else into the C + MPFR language with its rather peculiar requirements. Moreover, it’s either MPFR or an ODE library, unless we can find an ODE library written specifically for use with MPFR. The reason why can’t freely combine an ODE library with a finite-size arithmetic library is the same that prevents us from using the ODE-specific equation-evaluation procedures for other purposes: an ODE library does not contain ODE solver algorithms, but specific implementations of such algorithms that are less versatile than the algorithms themselves.

Leaving the narrow realm of development tools for numerical software, we could try to factor out the equations, ingredients 1 and 2, using a computer algebra system. Such a system lets us write down the equations as such, not only the numerical computation of its subterms. While the idea looks promising, the state of today’s computer algebra systems doesn’t make this a practically useful approach. They are not designed as parts of an ecosystem for scientific knowledge management. The formal languages they use for expressing terms and equations are insufficiently documented, and for commercial systems they are even partly secret. Some computer algebra systems have export functions that generate numerical code in a language like C or Fortran, but the exact semantics of this export are again opaque for lack of documentation.

7.1 Complex systems

For the example I have used in the last section, there is no real problem in practice because the whole model is rather simple. Ingredients 1 to 7 can be written down and composed on a single sheet of paper. We use computers only because the actual computation is very long to perform. It is quite feasible to do all theroretical work by hand, and write a simulation program just for doing the computation. That was in fact the dominant use of computers in science during their first few decades.

The situation changes drastically when we consider complex systems. If instead of the solar system we wish to simulate a protein at the atomic scale, we use a model that is overall very similar except for the second ingredient. Instead of Newton’s law of gravitation, a one-line formula, we have an expression for the interatomic forces made up of tens of thousands of terms. The list of these terms is constructed from the molecular structure by an algorithm, meaning that we need a computer – and thus formal languages – not only for simulating our model but already for defining it. The model itself is digital scientific knowledge.

Since we do not have adequate formal languages for writing down such digital models today, we cannot express them at all. We cannot analyze or discuss the
model, nor compare it in depth to competing models. All we have is research papers describing the design principles behind the model, and software written to perform a numerical evaluation. The software source code is impenetrable for anyone but its authors. Moreover, there is obviously no way to verify that the software evaluates the model correctly, because that would require some other expression of the model for comparison. This is again an instance of the problem that I discussed earlier for the definition of the semantics of programming languages. Our model should be part of the specification of our software, rather than being completely absorbed into its source code.

In the case of the popular models for biomolecular simulations, each of them is implemented by several different software packages, with each program producing somewhat different numbers for the same protein. On a closer look, each program actually implements its own variation of the model, with modifications made for performance reasons, or because the software authors believe the modification to be an improvement. In the end, what we think of as a model is really a family of different models derived from common design principles. In the absence of human-readable specifications of each variant, we cannot compile a detailed list of the differences, let alone estimate their impact on the results we obtain.

Similar situations exist wherever scientific models have become too complex to be written down on paper. As a second example, consider the Community Earth System Model [17], a popular model for the evolution of the Earth’s climate. One would expect such a model to consist of a large number of coupled partial differential equations describing the behavior of the atmosphere and the oceans, and their complex interactions. But it really is a software package that implements a numerical solver for the equations. Contrary to the situation in biomolecular simulation, a significant effort is made to ensure that this software package can be considered a reliable reference implementation. But even if we trust the software to reliably evaluate the model numerically, we have still lost all the non-numerical uses of a scientific model.

7.2 Software and data in computational science

It is customary in computational science to distinguish between computer programs, also called software, and the data that these programs process. But the above discussion of formal languages shows that this distinction between software and data is not fundamental. We could very well use a single language to define all aspects of a computation, and obtain the result in the same language. This is in fact very easy to do, by hard-coding all input data into the source code of the program. In today’s computing environments, that would be inconvenient in practice, but that is mostly due to the way our tools work.

From the point of view of digital knowledge management, it is desirable to identify the individual pieces of information we wish to handle, and the operations we wish to perform on them. The above analysis of a solar system
simulation provides a simple example. We would then design formal languages specifically as digital scientific notations for our knowledge items. Software tools would be just tools, consuming and transforming scientific knowledge but not absorbing it into its source code. In other words, all scientific knowledge would become \textit{data}.

Some recent developments can be seen as stepping stones towards this goal. I will mention a single example, the specification of differential equations in FEniCS [18]. FEniCS is a software package that solves partial differential equations numerically using the Finite Element method. A feature that distinguishes FEniCS from similar software packages is that it allows its users to write the differential equations to be solved in a notation very similar to traditional mathematics. In particular, the equations are written down as distinct information items, i.e. they are \textit{data}. They are not absorbed into program code that is structured according to the needs of software development. Similar approaches are used in other mathematical software packages. However, a crucial final step remains to be taken: Differential equations for FEniCS are written in a FEniCS-specific formal language that is not suitable for anything else than solving the equations in FEniCS. The scientific knowledge must be reformulated to fit to the tool. What we should have instead is a formal language for expressing all aspects of differential equations, and many tools, FEniCS being just one of them, that can process this formal language. In particular, we would like to be able to \textit{compose} differential equations describing some physical system from individual ingredients, much like the equations governing the solar system are composed from the law of motion and the law of gravity.

One psychological barrier to considering all scientific knowledge as data is the fact that scientific knowledge includes algorithms. In the example of the solar system simulation, the numerical method for solving Newton’s equation is an algorithm. The formal languages used to represent data in computational science do not permit the expression of algorithms. For most computational scientists, algorithms are parts of programs, and thus expressed in a programming language. However, it is easy to see that algorithms are just another kind of data. Compilers translate algorithms from one formal language to another, i.e. they process algorithms as data. The same can be said of many tools we use every day to develop and analyze software. The only novelty in my proposal is that algorithms that count as scientific knowledge should be available for all kinds of scrutiny \textit{in addition} to being executable by a computer.

We can also envisage an intermediate stage in which software tools continue to incorporate digital scientific knowledge just like they do today, but in which we also express and publish all digital scientific knowledge in human-friendly formal languages. The human-friendly version would then be part of the software’s specification, and the equivalence of the two formulations would be verified as part of software testing.
8 Human-computer interactions through formal languages

If computers are to be powerful tools for scientific research, the computer’s user interface must be designed to make the interaction of scientists with computers fluent and error-free. Whereas most other uses of computers happen through relatively simple interfaces (forms, graphical representations, command lines, ...), the interface between a scientist and a computer includes the formal languages in which scientific information is encoded for computation. In this respect, computational science resembles software development, where the human-computer interface includes programming languages. This similarity explains why techniques and tools from software engineering are increasingly adopted by computational science.

It is widely recognized in software engineering that software source code should be written primarily to explain the working of a program to other programmers, with executability on a computer being a technical constraint in this endeavor rather its main objective. Some authors even go farther and claim that the human understanding of the program, shared by the members of its development team, is the primary output of software development, because it is what enables the team to maintain and adapt the program as requirements evolve [19]. Software engineering research has therefore started to investigate the usability of programming languages by programmers [20].

In scientific research, human understanding takes an even more prominent role because developing an understanding of nature is the ultimate goal of science. Research tools, including software, are only a means to this end. Digital scientific notations are the main human-computer interface for research, and must be developed with that role in mind. The use of formal languages is a technical constraint, but suitability for research work and communication by human scientists must be the main design criterion.

Today’s digital scientific notations are programming languages and more or less well-defined file formats. In this section, I will outline the lessons learned from working with these adopted notations, and the consequences we should draw for the design of proper scientific notations in the future.

8.1 Human vs. computational semantics

A programming language fully defines the meaning of a program, and thus completely defines the result of a computation.\(^1\) However, software source code has a second semantic layer which matters only for human readers: references to conceptual domain knowledge in the choice of identifiers. Mismatches between

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\(^1\)At least it does in theory. The definitions of many popular languages are incomplete and ambiguous [21].
what a program does and what its source code suggests it does are a common source of mistakes in scientific software.

As an illustration, consider the following piece of Python code:

```python
def product(numbers):
    result = 1
    for factor in numbers:
        result = result + factor
    return result
```

To a human reader, the names `product`, `numbers`, and `factor` clearly suggest that this procedure multiplies a list of numbers. A careful reader would notice the `+` sign, indicating addition rather than multiplication. The careful reader would thus conclude that this is a multiplication program containing a mistake. This is exactly how a scientist reads formulas in a journal article: their meaning is inferred from the meaning of all their constituents, using an understanding of the context and an awareness of the possibility of mistakes.

For a computer, the procedure shown above simply computes the sum of a list of numbers. The identifiers carry no meaning at all; all that matters is that different identifiers refer to different things. As a consequence, the above procedure is executed without any error message.

If we analyze the situations that typically lead to program code like the above example, the careful human reader turns out to be right: most probably, the intention of the author is to perform multiplication, and the plus sign is a mistake. It is highly unlikely that the author wanted to perform an addition and chose multiplication-related terms to confuse readers.

Since the program is perfectly coherent from a formal point of view, approaches based on algorithmic source code analysis, such as type checking, cannot find such mistakes. Software testing can be of help, unless similar mistakes enter into both the application code and the test code. Of today’s software engineering techniques, the ones most likely to be of help are pair programming and code review. Like peer review of scientific articles, they rely on critical inspection by other humans.

Code review is also similar to peer review in that it is reliable only if the reviewer is an expert in the domain, even more so than the original author. In the case of software, the reviewer must have a perfect understanding of the programming languages and libraries used in the project. This is not obvious from the above example, which is particularly short and simple. Any careful reader will likely spot the mistake, even without much programming experience. But more subtle mistakes of this type do happen and do go unnoticed, in particular when advanced language features are used that perhaps even the code’s author does not fully understand. As an example, few scientists with basic Python knowledge are aware of the fact that the above five-line function could in fact compute almost anything at all, depending on the context in which it is used. All it takes is a
class definition that defines addition in an unexpected way.  

The main conclusion to draw from this is that digital scientific knowledge must be written in terms of very simple formal languages, in order to make human reviewing effective for finding mistakes. All the semantic implications of a knowledge item must be clear from the information itself and from the definition of the formal language it is written in. Moreover, a scientist working in the same domain should be able to read, understand, and memorize the language definition with reasonable effort and ideally pick it up while acquiring the domain knowledge itself, which is how we learn most of traditional scientific notation.

8.2 Flexibility in scientific notation

As I mentioned above, traditional pre-digital scientific notation is the result of an evolutionary process. In principle, scientists can use whatever notation they like, on the condition that they explain it in their publication. However, there is social pressure towards using well-established notation rather than inventing new ones. In practice, this leads to variable notation for new concepts that becomes more uniform over time as the concepts are better understood and consensus is reached about representing them in writing.

In contrast, formal languages used in computing are very rigid. The reasons are numerous and include technical aspects (ease of design and implementation) as well as historical ones (the advantages of flexibility were not immediately recognized). Perhaps the biggest barrier to flexibility left today is the near universal acceptance of rigidity as normal and inevitable, in spite of the problems that result from it. Most data formats used in computational science do not permit any variation at all. When data formats turn out to be insufficient for a new use case, the two possible choices are to “bend the rules” by violating some part of the definition, or to define a new format. Since bending the rules is often the solution of least effort in the short run, many data formats become ambiguous over time, with different software packages implementing different “dialects” of what everyone pretends to be a common format. Since computer programs lack the contextual background of humans, they cannot detect such variations, leading to an erroneous interpretation of data.

Programming languages are vastly more complex than data formats. In particular, implementing a programming language by writing of a compiler or interpreter is a significant effort, and requires competences that most computational scientists do not have. As a consequence, the programming languages used for computational science are few in number. Moreover, they are under the control of the individuals, teams, or institutions that produce their implementations. For all practical

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2Lest more experienced Pythonistas put up a smug grin reading this, I suggest they ask themselves if they fully understand how far the code’s result can be manipulated from the outside using metaclasses. I am the first to admit that I don’t.

3Readers familiar with computational structural biology have probably had bad surprises of this kind with the PDB [22] format.
purposes, computational scientists consider programming languages as imposed from outside. The only choice left to the individual scientist or team is which of the existing languages to use, and then work around its limitations.

A digital scientific notation should offer the same level of flexibility as traditional scientific notation: a scientist should be able to state “I use conventions X and Y with the following modifications”, defining the modifications in a formal language to make them usable by computers. Social pressure, e.g. in peer review, would limit abuses of this flexibility and lead to consensus formation in the long run.

8.3 References to the scientific record

The main infrastructure of science as a social process is the scientific record, which consists of the totality of scientific knowledge conserved in journal articles, monographs, textbooks, and electronic databases of many kinds. Scientists refer to the scientific record when they base new studies on prior work, but also when they comment on work by their peers, or when they summarize the state of the art in a review article, a monograph, or a textbook.

In scientific narratives, references to the scientific record are often imprecise by citing only a journal article, leaving it to the reader to find the relevant part of this article. It is, however, quite possible to refer to a specific figure or equation by a number. For computational work, references must be more precise: a dataset, a number, an equation. A digital scientific notation must therefore encourage the use of small information items that can be referenced individually while at the same time keeping track of their context. It matters that composite information items can be referenced as a whole but also permit access to their individual ingredients, as I have illustrated in my celestial mechanics example.

The rapidly increasing volume of scientific data and facts is creating a need for computer-aided analysis of the network of scientific knowledge. This has motivated the development of semantic publishing [2], which consists in publishing scientific findings in a machine-readable form where concepts become references to ontologies. Current research in semantic publishing focuses on giving machine-readable semantics to non-quantitative statements that are typically transmitted by the narrative of a journal article. The development of digital scientific notations that I wish to encourage by this essay can be seen as a variant of semantic publishing applied to computational methods. In this analogy, today’s scientific software is similar to today’s journal articles in that neither form of expression permits the automated extraction of embedded knowledge items.

9 Simple and flexible formal languages

The criteria exposed in the last section lead to a technical requirement for digital scientific notations: it must accommodate a large number of small and simple
formal languages and make it straightforward to define variants of them. This may well seem impossible to many computational scientists. Large, rigid, general-purpose languages are today’s standard for software development, whereas small, rigid, and undocumented languages dominate scientific data storage. However, there are examples of more flexible formal languages, which can serve as a source of inspiration for the development of digital scientific notations. I will describe two of them in this section.

The main technical obstacle to flexibility in formal languages is the requirement for composition that I have discussed earlier: the information items that enter into a composition must all be expressed in the same language. If that condition is not satisfied, an additional effort must be invested in the form of language conversion or more complex software that can process multiple languages.

The solution is to design a framework for a family of formal languages, and develop generic tools that can process any member of this family and also compositions of different members. In other words, flexibility enters the design and the support infrastructure at a very early stage. This principle should become clearer from two concrete examples: XML and Lisp.

9.1 XML: composable data formats

XML [11] is a framework for defining formal languages that express tree-structured data. The central concept in XML is the element, which is a node in a tree whose type is identified by a tag. The tag also defines which attributes the element can have, and which conditions its child elements must satisfy. A concrete XML-based data format is defined by a schema, which contains an exhaustive list of the allowed tags and the constraints on the element types defined by each tag. Given a data file and the schema it is supposed to respect, generic XML processing tools can validate the data file, i.e. check that it conforms to the schema, and also perform many types of data transformation that do not depend on the semantics of the data. Finally, writing programs that do semantics-dependent processing is facilitated by support libraries that take care of the semantics-independent operations, in particular parsing and validating the incoming information and producing correct result files. Because of these advantages, XML has become very popular and a large variety of schemas has been defined. Examples that may be familiar to computational scientists are MathML and OpenMath for mathematical formulas, SVG for vector graphics, CML for chemical data, and SBML for systems biology.

Composition of XML data means constructing a tree from elements defined in different schemas. This was made possible with the introduction of XML namespaces. A single-schema XML document starts with a reference to its schema. A multi-schema XML document lists multiple schemas and associates a unique name with each of them. That name is then prefixed to each tag in the document. This prefix ensures that even in the presence of tag homonyms in
the document’s schemas, each element has a unique and well-defined tag.

The XML namespace mechanism is an implementation of the superlanguage approach that I have described earlier. Processing such superlanguages is made straightforward because the mechanisms for defining them are part of the XML definition. All modern XML processing software implements namespaces, and therefore can handle arbitrary superlanguages inside the XML universe.

XML namespaces are not a magical solution to composing unrelated data items. Any software that performs semantics-dependent processing still needs to deal with each schema individually. But the tasks of defining languages, processing them, and processing compositions are enormously simplified by the XML framework. Defining an XML schema is much simpler than designing a complete data format, let alone a data format open for extensions. Processing someone else’s XML data is also much simpler than processing someone else’s ad-hoc data format, because the schema provides a minimum of documentation. Finally, the namespace mechanism encourages the definition of small schemas that can then be composed, making well-designed XML-based data formats easier to understand for human readers.

9.2 Lisp: extensible programming languages

Most programming languages used today are constructed in much the same way. A syntax definition specifies which sequences of text characters are legal programs. This syntax definition is set in stone by the language designer. Some syntactical elements define fundamental data types, others fundamental executable operations. These basic building blocks can be combined by the programmer into larger-scale building blocks using language constructs for defining data structures, procedures, classes, etc. In fact, programming is almost synonymous with defining such entities and giving them names for later referring to them. In other words, programming means extending the language by new building blocks, the last of which is the program to be run. The programmer cannot modify the syntax in any way, nor take any features away from the basic language. This means in particular that the programmer cannot make the language any \textit{simpler}.

One of the oldest family of programming languages, the Lisp family, differs from this picture in an important way. Its syntax is defined in two stages. The first stage merely defines how a central data structure called a \textit{list} is written in terms of text characters. The elements of a list can be any basic Lisp data type, e.g. numbers or symbols, but also other lists. Nested lists are equivalent to trees, and in fact Lisp’s nested lists are very similar to the trees of elements that I have described in the section on XML. The second stage of Lisp’s syntax defines which lists are legal programs. The general convention is that the first element of a list specifies a language construct to which the remaining elements are parameters. For example, the list \((+ \ 2 \ 3)\) means “perform the + operation
on the numbers 2 and 3”, whereas the list \(\text{define x (+ 2 3)}\) means “set variable x to the value of the expression defined by the list \((+ 2 3)\)”.

This two-stage syntax is exploited in what is a very rare feature in programming languages: the second syntax level can be modified by the programmer, using a language construct called a \textit{macro}. Technically, a macro is a function called as part of the compilation of Lisp code. When the compiler hits a list whose first element specifies a macro, it runs the macro function and substitutes the original macro-calling list by the macro function’s return value, which is then compiled instead.

To understand the power of this construct, consider that a compiler is a program that transforms another program written in language A into an equivalent program written in language B. That is exactly what a macro does: it translates a program written in some language M into basic Lisp. The language M is defined by the macro itself, just like any compiler is an operational definition of a language as I explained before. Whatever the macro accepts as arguments is a valid program in M. A macro thus \textit{is} a compiler, and by defining macros a programmer can define his or her own languages with no other restrictions than respecting the top layer of Lisp’s syntax, i.e. the list syntax. Most macros merely define small variations on the basic Lisp language, but nothing stops you from writing a \texttt{fortran} macro to implement a language equivalent to Fortran except that its syntax is defined in terms of nested lists.

The use of macros as building blocks of compilers has been pushed to a very advanced level in the Racket language [23], a Lisp dialect which its developers describe as a “programmable programming language”. The path from the first Lisp macros of the 1960s via Scheme’s hygienic macros to today’s Racket has been a long one. For example, it turned out that making macros composable is not trivial [24]. Today’s Racket programming environment contains a large number of languages for various purposes. Plain “racket” is a standard general-purpose programming language. A core subset of “racket” is available as “racket/base”. Several languages are simplified forms of “racket” for teaching purposes. The simplification does not merely take out language features, but exploits the gain in simplicity for providing better error messages. Other languages are extensions, such as “typed/racket” which adds static type checking. But Racket also lifts the traditional Lisp restriction of list-based syntax, providing a mechanism to write language-specific parsers. Both Java and Python have been implemented in Racket in this way. A language definition in Racket is nothing but a library [25], meaning that any number of languages can co-exist. Moreover, a new language can be based on any existing one, making it straightforward to define small modifications.

A big advantage of the Lisp/Racket approach to implementing new languages is that all those languages are interoperable, because they are all compiled to basic Lisp/Racket. This is an implementation of the translation approach to composing different languages that I have described before. Another advantage is that defining new languages becomes much easier. Implementing a big language
such as Python remains a difficult task even in Racket. But implementing a small variation on an existing language – take away some parts, add some others – is simple enough to be accessible to an average software developer.

10 Designing digital scientific notations

The main conclusion from the analysis that I have presented in this essay is that digital scientific notations should be based on formal languages with the following properties:

- **Small and simple**: each formal language must be so small and simple that a scientist can memorize it easily and understand its semantics in detail.

- **Flexible**: a scientist must be able to create modifications of existing languages used in his/her field in order to adapt them to new requirements and personal preferences.

- **Interoperable**: composition of digital knowledge items expressed in different languages must be possible with reasonable effort.

The two examples I have presented above suggest that a good approach is to define a framework of languages and implement generic tools for common manipulations. The foundation of this framework should provide basic data types and data structures:

- numbers (integers, rationals, floating-point, machine-level integers)
- symbols
- text
- N-dimensional arrays
- trees
- sets
- key-value maps (also called associative arrays, hash tables, or dictionaries)

The representation of these fundamental data types in terms of bit sequences can be based on existing standards such as XML (text) or HDF5 (binary). It is probably inevitable to have multiple such representations to take into account conflicting requirements of different application domains. As long as automatic loss-less interconversion can be ensured, this should not be an obstacle to interoperability. An added advantage of keeping the lowest level of representation flexible is the possibility to adapt to future technological developments, for example IPFS [26] whose “permanent Web” approach seems well adapted to preserving the scientific record.
There should also be a way to represent algorithms, but it is less obvious how this should best be done. Any of the common Turing-complete formalisms (lambda calculus, term rewriting, ...) could be used, but it may turn out to be useful to have access to less powerful formalisms as well, because they facilitate the automated analysis of algorithms.

A next layer could introduce domain-specific but still widely used data abstractions, e.g. from geometry. For much of mathematics, the OpenMath content dictionaries [27] could be adopted. On top of this layer, each scientific community can build its own digital scientific notations, and each scientist can fine-tune them to specific needs.

An illustration of how these principles can be applied is given by the MOlecular SimulAtion Interchange Conventions (MOSAIC) [28], which define a digital notation for molecular simulations. MOSAIC lacks the common layer of data types listed above, and is therefore not easily interoperable with other (future) digital notations. It does, however, define data structures specific to molecular simulations in terms of more generic data structures, in particular arrays. MOSAIC defines two bit-level representations, based on XML and HDF5. A Python library [29] proposes three further implementations in terms of Python data structures, and implements I/O to and from the XML and HDF5 representations.

Traditional scientific notations have evolved as a byproduct of scientific research, and digital scientific notations will have to evolve in the same way in order to be well adapted to the task. In this spirit, the ideas listed in this section are merely the basis I intend to use in my own future work, but they may well turn out to be a dead end in the long run. I would like to encourage computational scientists to develop their own approaches if they think they can do better. As I have stated in the introduction, my goal with this essay is not to propose solutions, but to expose the problem. If computational scientists start to think about “digital scientific notation” rather than “file formats” and “programming languages”, I consider my goal achieved.

References


